A Time-Frequency Analysis of Oil Price Data

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Purpose and summary

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- Challenge: to understand oil price dynamics in the long run, but also on short scales, can variations on different scales be understood in uniform way?
- Idea: to adapt and exploit elaborated tools developed for turbulence data.

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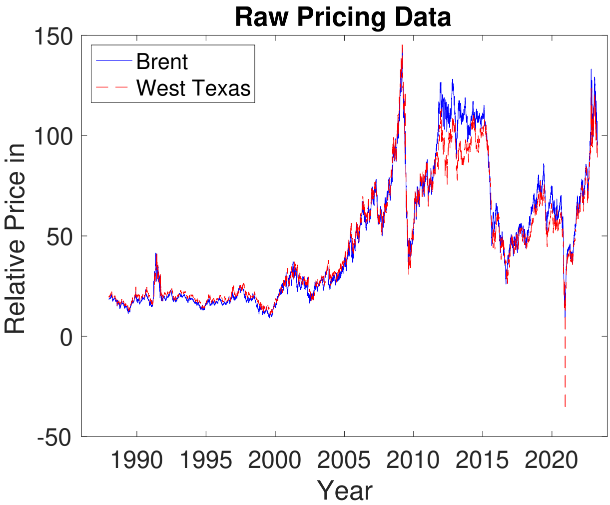
Summary:

- Oil price data have a rich multiscale structure that may vary over time.
- The monitoring of these variations shows regime switches.
- The quantitative analysis is carried out by a wavelet decomposition method.

The data set



Oil price data from 1987 to 2022 for West Texas (red dashed line) and Brent (solid blue line).

Standard price model

- Price $P(t) =$ a drift $d(t)$ + a diffusion, that can be expressed in terms of a Brownian motion $B(t)$ and volatility σ_t :

$$\frac{dP(t)}{P(t)} = d(t)dt + \sigma_t dB(t)$$

- The Brownian motion B is a Gaussian process with independent and stationary increments:

$$\mathbb{E}\big[(B(t+\Delta t) - B(t))^2\big] = |\Delta t|$$

It is self-similar:

$$B(at) \overset{dist.}{\sim} a^{1/2} B(t) \quad \text{for any } a > 0$$

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- The volatility is the most important ingredient of the standard model.
  - When $\sigma_t \equiv \sigma$ and $d(t) \equiv d$, this is the Black–Scholes model (1973).
  - When $\sigma_t \equiv \sigma(t, P(t))$, this is the local volatility model (Dupire, 1994; Derman and Kani, 1994).
  - When σ_t is a stochastic process, this is the stochastic volatility model (Hull and White, 1987; Heston, 1993).

Multi-fractional price model

$\rightarrow$ Motivated by the data, let increments of multi-fractional Brownian motion model “relative” price changes, essentially:

$$R_{n+1} = \frac{P(t_{n+1}) - P(t_n)}{P(t_n)} = \sigma_n(B_{H_n}(t_{n+1}) - B_{H_n}(t_n)).$$

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- Price $P(t)$:

$$\frac{dP(t)}{P(t)} = d(t)dt + dB_{H,\sigma}(t)$$

  where $B_{H,\sigma}$ is a multi-fractional process (H_t and σ_t are time-dependent) (Lévy-Véhel 1995).
- If $H_t \equiv H$ and $\sigma_t \equiv 1$, then $B_{H,\sigma} = B_H$ fractional Brownian motion.

→ Some issues:

- Rapid Monte-Carlo simulation of price "paths".
- Estimation of the parameters.
  ↪ use of wavelets.

Radical change of perspective: Hurst coefficient and volatility are the fundamental quantities.

The Hurst coefficient governs the scaling of volatility of increments as function of time lag.

Example: If we condition "volatility" to be one at time lag 1 (say in annualized units), then St.dev($\sigma B^H(t) = t^H$), $t \in (0,2), \sigma = 1$:

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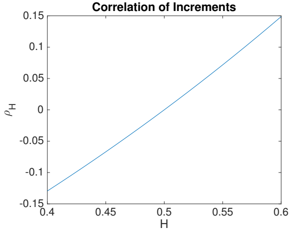

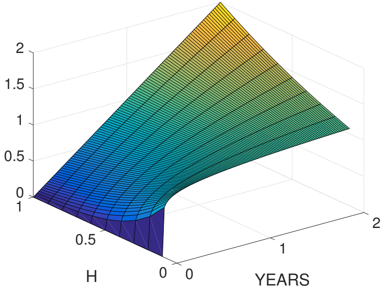

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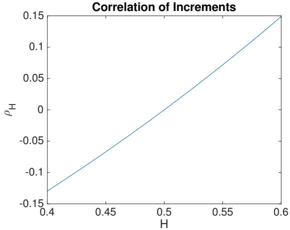
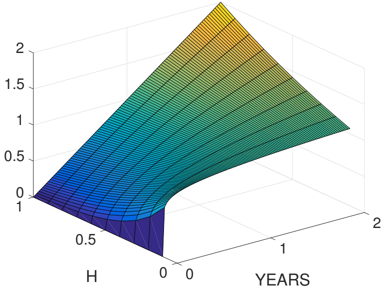




Classic case:

$H = 1/2$: independent increments.

Limit cases:

$H \to 1$: Increments equal: $\Delta B_n = \Delta B_{n-1}$.

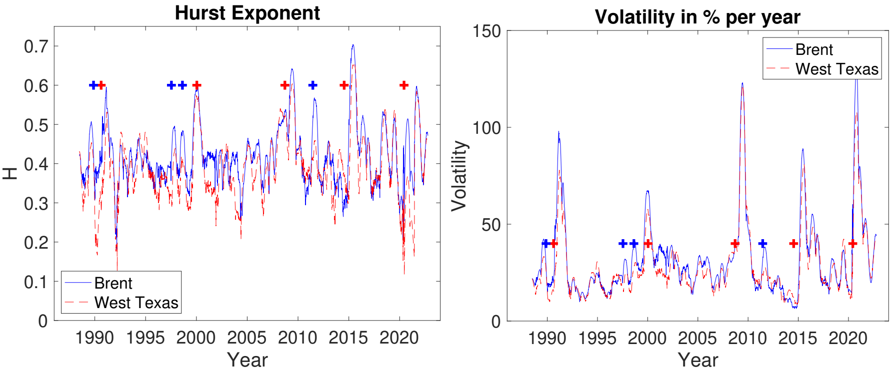
$H \to 0$: Increments alternate in sign: $\Delta B_n = -\Delta B_{n-1}$.

Wavelets

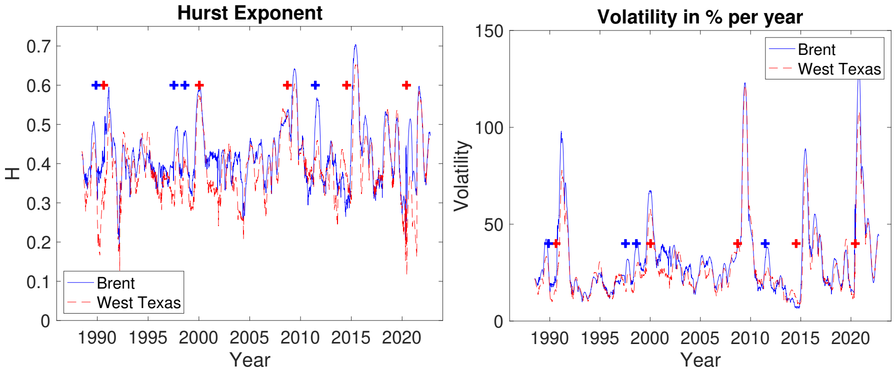
- Main context: Signals may have frequency content that varies with time. Ex: speech.
- A Fourier decomposition gives the "global" frequency decomposition.
- A wavelet decomposition gives a local characterization of the frequency contents.

→ To detect changes in the multiscale character of the prices the wavelets are useful.

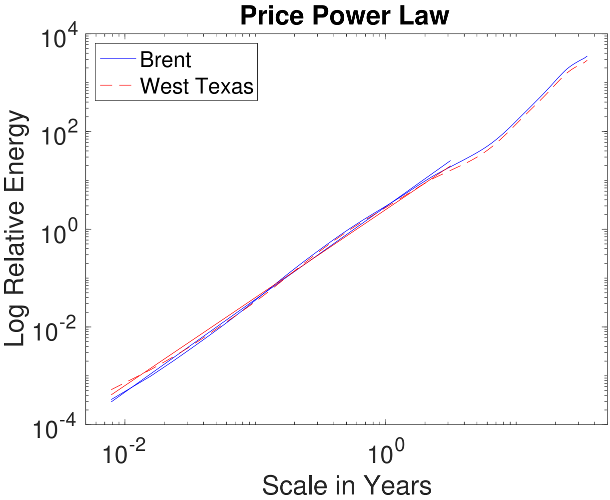
Cf: Meyer 1984, Mallat, Daubechies...



Hurst Exponent
H
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1990 1995 2000 2005 2010 2015 2020
Year
Brent
West Texas
Volatility in % per year
Volatility
150
100
50
0
1990 1995 2000 2005 2010 2015 2020
Year
Brent
West Texas

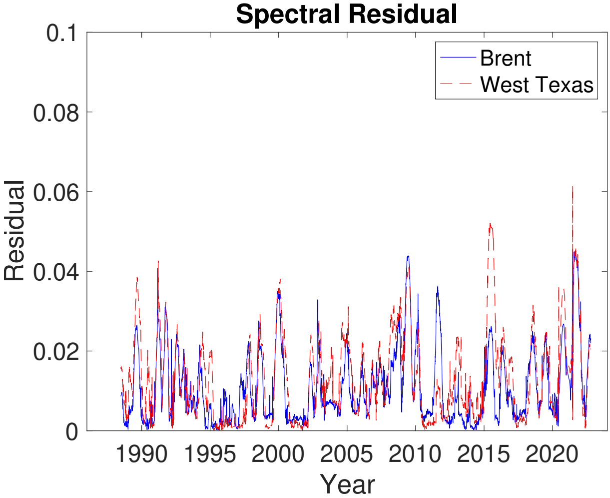


August 1990; January 2000; September 2008; July 2014; March 2020
Kuwait Invasion; Y2K; Lehman; Liquidation derivatives; Covid.
November 1989; July 1997; August 1998; September 2010
Berlin Wall; Asian financial crisis; Russian financial crisis; European debt crisis.



The "global power law" for West Texas data (red dashed line) and Brent data (blue solid line).

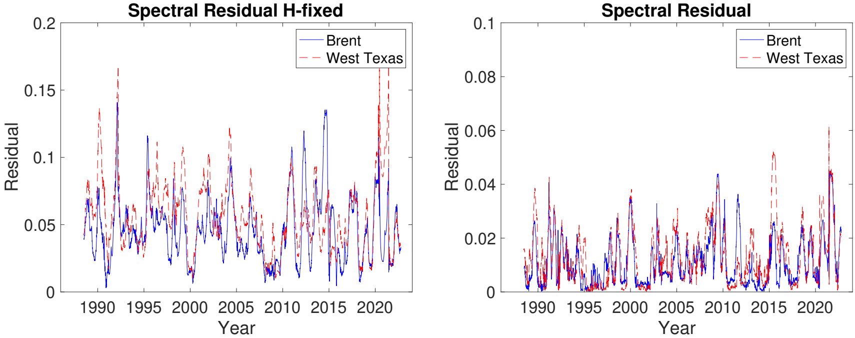
- A global power law (with $H = .45$ & $.40$) is consistent with a situation in which the Hurst exponent and volatility vary over subsegments.



Spectral misfits for the West Texas data (red dashed line) and the Brent data (solid blue line).

- The spectral misfits are low and statistically homogeneous with respect to time.

Comparison with standard model



Spectral misfits

Left when we condition on $H = 1/2$ to enforce uncorrelated returns.

Right with the multi-fractional model.

- With H fixed, the spectral misfits are relatively high; they also vary significantly during the special periods.

Conclusions

- Oil data contain multiscale fluctuations different from (log) normal diffusions.
- Special periods with $H_t > 1/2$ can be identified.
- Standard volatility estimates are not appropriate during the special periods when $H_t > 1/2$.
- Regime changes and special epochs revealed via changes in the multifractal parameters.
- Rough versus smooth price paths reflects behavior of market participants.
- Price in recent epoch high, but the dynamics not abnormal.