# Modelling and understanding human longevity: What can be learned from population dynamics?

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Based on joint works with N. El Karoui, H. Labit-Hardy and S. Arnold

# Outline

# Longevity risk

- Population ageing and uncertainty around future longevity developments are producing multiple challenges for governments and private actors:
  - Sustainability of pay-as-you-go public pension systems.
  - Longevity risk management for insurers and funded pension systems: Increased regulatory capital (Barrieu et al.(2012)).

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- Data and mortality models used for decision making in state pension and public health reforms, regulatory reserving policies, longevity financial products...
- Important tradition of mortality data collection:
  - Multiple available databases (National statistical institutes, UN, WHO, HMD, ...).

# Traditional mortality modelling

- Classical tool for modelling and forecasting human longevity:

  Age-specific mortality rates.

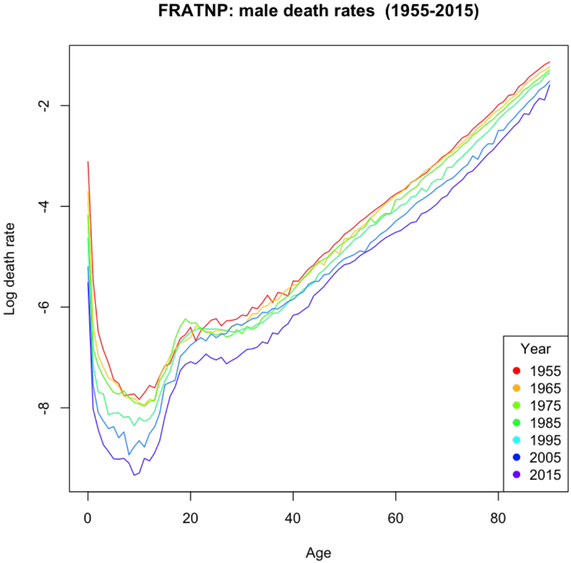
- Standard mortality models: parametric models estimated from the data.
- Gompertz model (1825):

$$\mu(a) = \alpha e^{\beta a}.$$

- Now there are multiple models used for mortality modelling and forecasting:

  Lee-Carter (1992), Renshaw and Haberman (2006), Cairns, Blake and Dowd (2006), Ludkovski, J Risk, Zai (2018)...

Figure: French male log death rates, 1950-2015




(source: HMD)

# Challenges

Observed mortality is a by-product of population dynamics (not taken into account in standard models):

↳ Result of complex demographic and social mechanisms.

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**Limitations** of only studying age-specific mortality rates:

- Not possible to measure impact of macro environment.
  - ↳ Ex: Importance of macro public health measures.
- Impact of the population not taken into account:
  - ↳ Aggregation issues.
  - ↳ Impact of population, cohort size, interactions?
- Need for finer-grained population dynamics models in the presence of heterogeneity (longevity varies with individual characteristics).

# Socioeconomic gradient in mortality

*How can a cause-of-death reduction be compensated for by the population heterogeneity? A dynamic approach, with H. Labit Hardy, S. Arnold and N. El Karoui, IME.*

- Research on the relationship between socioeconomic status (SES) and mortality is longstanding (Villermé (1830), General Register Office (1851))
  ⇒Consensus on the strong correlation between SES and mortality.
- New trends observed in the past decade: increase in socioeconomic and geographical gaps in health and mortality.
  - Ex: Gap in male life expectancy at 65 between higher managerial and routine occupations (England Wales): 2.4 years 1982-1986, 3.9 years 2007-2011).
  - National Research Council Report (2011) on diverging trends in longevity.

# Taking heterogeneity into account

- Not taking into account heterogeneity can lead to:
  - Increased inequalities due to public health reforms (Alai et al. (2017)) or "unfair" redistribution properties of pension systems (Holzmann et al. (2017)).
  - Errors in funding of annuity and pension obligations (Meyricke and Sherris (2013), Villegas and Haberman (2014)).
- Better understanding of heterogeneity allows for a better understanding of the basis risk (Longevity basis risk report (2014)).

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More and more data released by international organizations and national statistical institutes ⇒ new issues can be investigated.

# Modelling heterogeneous mortality rates

- Growing literature on the joint modelling and forecasting of the mortality of socioeconomic subgroups Bensusan (2010), Jarner and Kryger (2011), Villegas and Haberman (2014), Cairns et al. (2016) ...
- Bringing many challenges:
  - **Consistency** of sub-national and national estimates/forecasts (Shang and Hyndman (2017), Shang and Haberman (2017)).
  - Interpreting targets set by institutions (Department of Health, WHO) (Alai et al. (2017)).

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**Our approach:** to take into account all population data.

*How do changes in the socioeconomic composition of the population affect aggregated indicators? Could we miss a cause-of-death reduction in the presence of heterogeneity?*

# Outline

# Data

- **Two datasets**:
  - 1981-2007: Department of Applied Health Research, UCL.
  - 2001-2015: Office for National Statistics, UK.
- English cause-specific number of deaths and mid-year population estimates per socioeconomic circumstances, age and gender.

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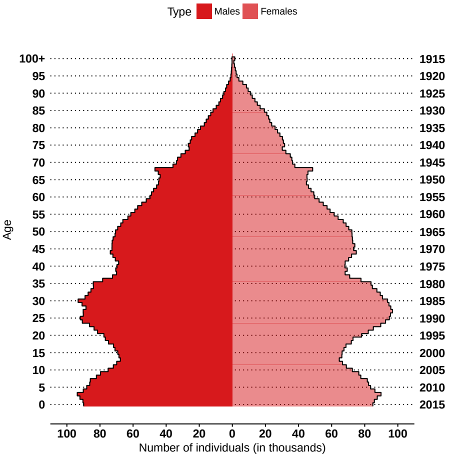
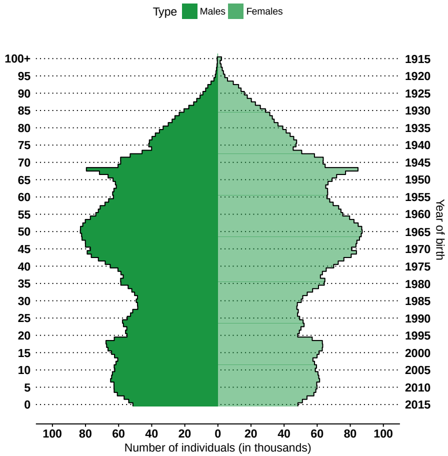
Socioeconomic circumstances are measured by the Index of multiple deprivation (IMD), based on individuals' postcodes.

- Small areas (LSOA) are ranked based on seven broad criteria: income, employment, health, education, barriers to housing and services, living environment and crime.
- This ranking makes it possible to divide the population into 5 quintiles with about the same number of individuals in each quintile.

# Age-pyramids by IMD quintile, 2015

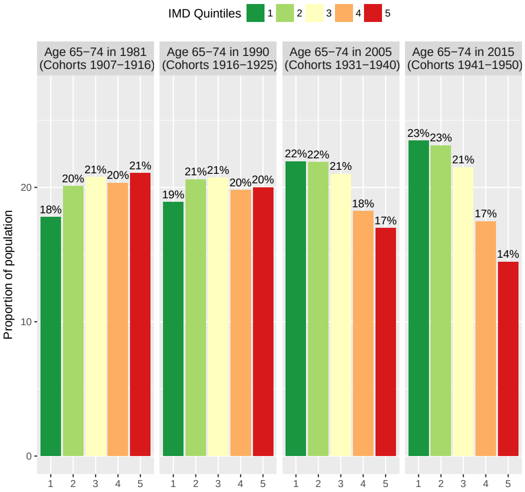



(a) Most deprived quintile ($)
Median age: 33y



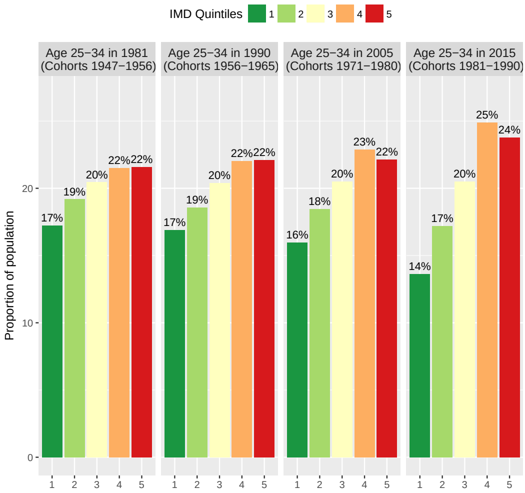

(b) Least deprived quintile ($$$$$)
Median age: 44.2y

Figure: Composition of males age class **65-74** in *years* **1981, 1990, 2005, 2015**.




- Decrease in deprivation over time for older age classes. (IMD 1+2: 28% → 46%).

Figure: Composition of males age class **25-35** in *years* **1981, 1990, 2005, 2015**.




- Increase in deprivation for younger age classes. (IMD 1+2: 36% → 31%).

# Heterogeneous population dynamics

- Simple age-structured population dynamics framework to illustrate different impacts of heterogeneity on the aggregated mortality.
- **Deterministic** evolution of each subgroup is described by a McKendrick (1926) -Von Foerster (1959) time dependent model.

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- Equation for each gender $\epsilon = m$ or $f$ and subgroup:
  - Ageing law:

$$(\partial_a + \partial_t) g_j^\epsilon(a,t) = -\mu_j^\epsilon(a,t) g_j^\epsilon(a,t)$$

  - Birth law:

$$g_j^\epsilon(0,t) = \int_0^{a^\dagger} p^\epsilon g_j^f(a,t) b_j(a,t) da$$

  - Initial Pyramid:

$$g_j^\epsilon(a,0)$$

# Aggregated population

- Aggregated population:
  - $g^{\epsilon}(a,t) = \sum_{j=1}^{p} g_j^{\epsilon}(a,t)$
  - Ageing law: $(\partial_a + \partial_t) g^{\epsilon}(a,t) = -d^{\epsilon}(a,t) g^{\epsilon}(a,t)$
- Aggregated death rate:
  - Weighted sum of the subpopulations death rates:

$$d^{\epsilon}(a,t) = \sum_j w_j^{\epsilon}(a,t)\mu_j^{\epsilon}(a,t), \quad w_j^{\epsilon}(a,t) = \frac{g_j^{\epsilon}(a,t)}{g^{\epsilon}(a,t)} \tag{1}$$

  - $d$ depends non-linearly on the population inputs: $g_j^0$, $\mu_j$, and $b_j$.
- Even with time-independent rates $\mu_j^{\epsilon}(a, \not{t})$

$\Rightarrow$ the aggregate death rate $d^{\epsilon}(a, \mathbf{t})$ depends on time, due to changes in the composition of the heterogeneous population.

# Numerical results

- **Goal**: To use the population dynamics model in order to analyse different impacts of heterogeneity on the aggregated mortality.
- **Two applications**
  1. Impact of the age-pyramid heterogeneity.
     ↳ Compare order of magnitude of mortality changes induced by compositional changes to constant mortality improvements.
  2. Cause specific mortality reduction vs "reverse" cohort effect.
     ↳ Compensation of cause-specific mortality reduction due to adverse compositional changes in some cohorts.

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- We consider a synthetic population composed of the most and least deprived IMD quintile (for illustrative purposes).

# Three demographic scenarios

**A** **Scenario A**: Population evolution with time-invariant mortality

Compositional changes isolated ⇒ death rates in each subpopulation do not depend on time:

$$d^{\epsilon}(a,t) = \mu_1^{\epsilon}(a) w_1^{\epsilon}(a,t) + \mu_5^{\epsilon}(a) w_5^{\epsilon}(a,t).$$

**B** **Scenario B**: Population evolution with mortality improvement

Constant annual mortality improvement rates of $r = 0.5\%$:

$$d^{\epsilon}(a,t) = \mu_1^{\epsilon}(a)(1-r)^t w_1^{\epsilon}(a,t) + \mu_5^{\epsilon}(a)(1-r)^t w_5^{\epsilon}(a,t).$$

**C** **Scenario C**: Mortality improvements without composition changes

$$d^{\epsilon}(a,t) = \mu_1^{\epsilon}(a)(1-r)^t w_1^{\epsilon}(a) + \mu_5^{\epsilon}(a)(1-r)^t w_5^{\epsilon}(a).$$

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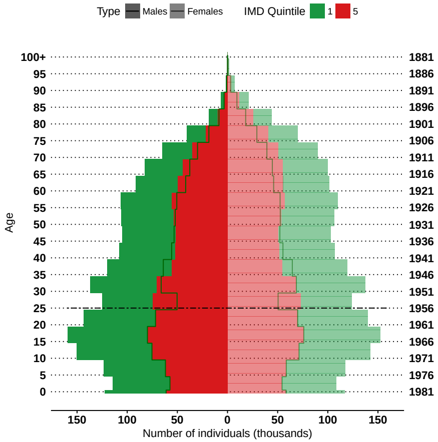
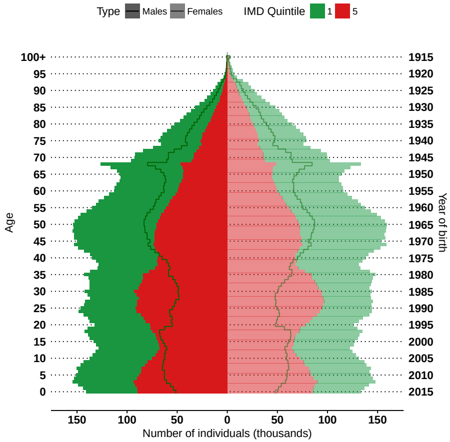
Mortality rates and initial age-pyramid fitted to the data for years 1981 and 2015.

# Initial age pyramids



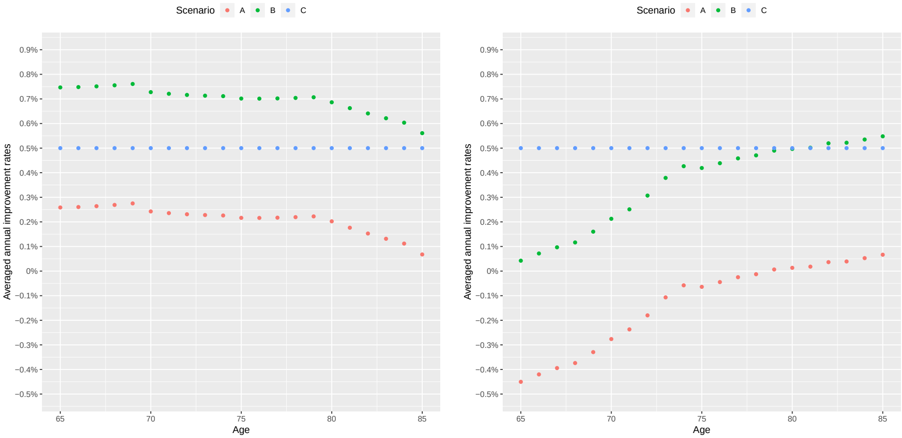

(a) 1981 Inputs




(b) 2015 Inputs

# Mortality improvement rates (males)

Figure: Average annual mortality improvement rates over years 0-30




(a) 1981 Inputs (b) 2015 Inputs

# Summary

- 1981 initial population: positive contribution from changes in the composition of the 65+ age class.
- 2015 initial population: negative contribution from composition changes ⇒ might offset future mortality improvement rates.
- Order of magnitude of age-pyramid heterogeneity impact can represent 0.2%- 0.5% in annual mortality improvement rates.

# Composition changes and compensation effect

**Example** of scenario illustrating impact of changes of demographic rates:

- Cause-specific reduction of mortality vs "reverse" cohort effect (adverse composition changes quantified by changes in birth patterns).
- Difficulty in interpreting the data at the aggregated level when coupled changes of different nature occur.

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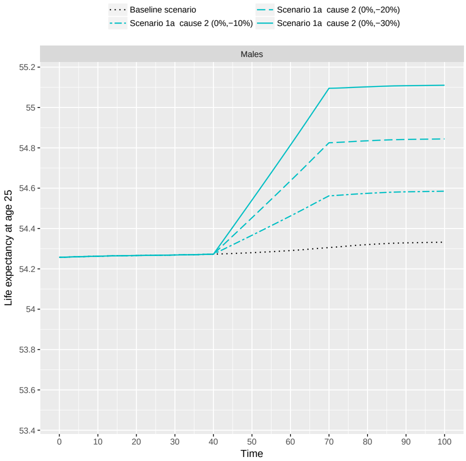
- Comparison with **Baseline ("neutral") scenario**:

  Constant demographic rates and population composition.
- Indicator: **Period life expectancy** at 25 (average lifetime remaining for an imaginary individual living in the mortality conditions of year $t$).

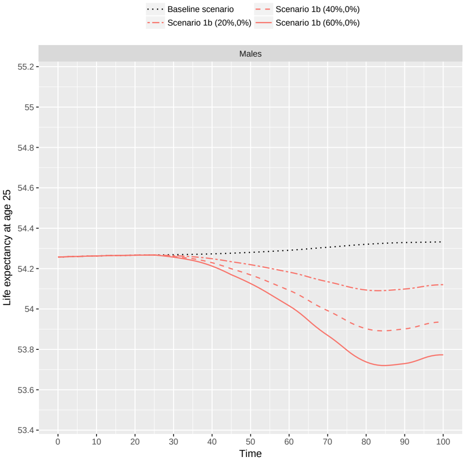
# Scenario 1a: Cause of death reduction

Figure: Reduction in mortality rates from cardiovascular disease (CVD)




- Reduction of 10, 20 and 30% over a period of 30 years, starting at $t = 40$.

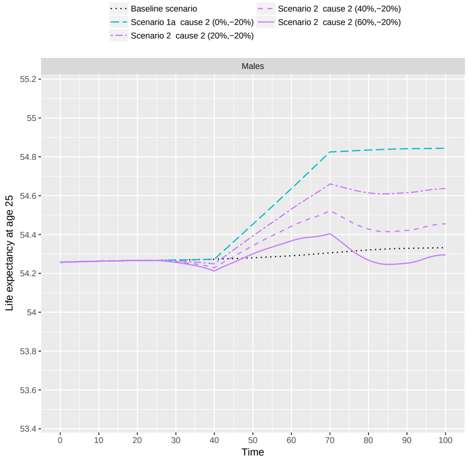
# Scenario 1b: "Reverse" Cohort effect




- Reverse cohort effect:

  Increase in birth rates in most deprived subgroup over period $[0, 20]$.
- $\nearrow$ 60% $\Rightarrow$ cohorts composed of 63% of most deprived subgroup.

# Scenario 2: Combined CoD reduction and cohort effect




*When the population heterogeneity is not taken into account, cause-of-death mortality reduction could be compensated for and/or misinterpreted depending on the population composition evolution.*

# Outline

# Birth-Death-Swap processes

*Pathwise construction of Birth-Death-Swap systems leading to an averaging result in presence of two timescales, with N. El Karoui*

- **Goal**: To study the random evolution of an heterogeneous population including:
  - A time-varying random environment.
  - Model changes in the population's composition induced by interacting individuals changing characteristics.

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- Main contributions:
  - General mathematical framework and tools to study such processes.
  - Study of the aggregated "macro" dynamic produced by such models.
- **Averaging result**: aggregated mortality rates are approximated by "averaged" rates depending non-trivially on the number of individuals in the population.

Thank you for you attention!