The climate-extended credit risk model

Josselin Garnier (Ecole Polytechnique, Lusenn)
Anne Gruz (Iggaak)

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Climate-Extended Risk Model (CERM)

- Objectives:
  - Determine the loss distribution of a a credit portfolio.
    $\rightarrow$ look for the expected and unexpected losses (expectation and quantile).
  - Propose a credit risk model which extends the model defined by the Basel Committee to climate (physical and transition) risks.
- Ingredients:
  - Credit/climate rating, (IPCC) scenarios.
  - Initial loan distribution, reloading of outstanding loans.
- Results:
  - Measure the incremental cost of risk and capital to inform credit allocation decisions.
  - Optimize the overall climate strategy, including financing existing clients' adaptation/mitigation plans and shifting assets to green lenders and green collateral.

Loss for a credit portfolio

- Goal: assess the loss of a credit portfolio.
- The loss of the portfolio L is the sum of the random losses of the borrowers.
  $\hookrightarrow$ L is random.
- The expected loss $L^{\mathrm{e}} = \mathbb{E}[L]$ of the portfolio is the sum of the expected individual losses.
- The unexpected loss is a quantile (value at risk) L^{u} of the loss of the portfolio:
  $$\mathbb{P}(L \leq L^{\mathrm{u}}) = 0.99 \text{ (or } 0.999 \text{ or } 0.9)$$
  The quantile of a sum is not the sum of the quantiles.
  $\hookrightarrow$ A model is needed for the dependence structure.

Expected loss of the portfolio

- For the qth borrower, the expected loss ($\mathrm{EL}^{(q)}$) can be expressed in terms of probability of default ($\mathrm{PD}^{(q)}$), loss given default ($\mathrm{LGD}^{(q)}$), exposure at default ($\mathrm{EAD}^{(q)}$):

$$\mathrm{EL}^{(q)} = \mathrm{PD}^{(q)} \times \mathrm{LGD}^{(q)} \times \mathrm{EAD}^{(q)}$$

- The expected loss $L^{\mathrm{e}} = \mathbb{E}[L]$ of the portfolio
  - is the sum of the expected individual losses,
  - can be expressed by grouping the borrowers:

  The borrowers belong to different groups $g = 1, \dots, G$, that represent
  - geographic regions,
  - economic sectors,
  - climate risk mitigation and adaptation strategies,
  - collateral types.

  The borrowers have different ratings $i = 1, \dots, K-1$ at time 0 (the rating K is default).

Expected loss of the portfolio

Expected loss L^{e} at the time horizon T:

$$L^{\mathrm{e}} = \sum_{t=1}^{T} L_t^{\mathrm{e}},$$

$$L_1^{\mathrm{e}} = \sum_{g=1}^{G} \sum_{i=1}^{K-1} (\boldsymbol{M}_{g,1})_{iK} \mathrm{LGD}_{g,i,1} \mathrm{EAD}_{g,i,1},$$

$$L_t^{\mathrm{e}} = \sum_{g=1}^{G} \sum_{i,j=1}^{K-1} \big(\boldsymbol{M}_{g,1} \cdots \boldsymbol{M}_{g,t-1}\big)_{ij} (\boldsymbol{M}_{g,t})_{jK} \mathrm{LGD}_{g,j,t} \mathrm{EAD}_{g,i,t}, \text{ for } t \geq 2.$$

Here

- $\boldsymbol{M}_{g,t}$: unconditional $K \times K$ migration matrix,
- $\mathrm{EAD}_{g,i,t}$: Exposition At Default, total exposure at default (in case of default at time t) for all borrowers in group g and with initial rating i,
- $\mathrm{LGD}_{g,j,t}$: Loss Given Default,

depend on group $g \in \{1, \dots, G\}$ and time $t \in \{1, \dots, T\}$.

Expected loss of the portfolio: Migration matrix

initial rating	credit rating at year-end							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	0,9112	0,0800	0,0070	0,0010	0,0005	0,0001	0,0001	0,0001
AA	0,0070	0,9103	0,0747	0,0060	0,0010	0,0007	0,0002	0,0001
A	0,0011	0,0234	0,9154	0,0508	0,0061	0,0026	0,0001	0,0005
BBB	0,0002	0,0030	0,0565	0,8798	0,0475	0,0105	0,0010	0,0015
BB	0,0001	0,0010	0,0055	0,0777	0,8177	0,0795	0,0085	0,0100
B	0,0000	0,0005	0,0025	0,0045	0,0700	0,8350	0,0375	0,0500
CCC	0,0000	0,0001	0,0010	0,0030	0,0259	0,1200	0,6500	0,2000
Default	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	1,0000

1-year migration matrix with $K = 8$. Each row corresponds to an initial rating. Each column corresponds to a rating at the end of one year. As "Default" is absorbing, the last line is of the form $(0, \ldots, 0, 1)$.

Unexpected loss of the portfolio: The need for a credit risk model

- The expected loss $L^{\mathrm{e}} = \mathbb{E}[L]$ of the portfolio is the sum of the expected individual losses.
- The unexpected loss is a quantile L^{u} of the loss of the portfolio:

$$\mathbb{P}(L \leq L^{\mathrm{u}}) = 0.999 \text{ (or } 0.99 \text{ or } 0.9)$$

The quantile of a sum is not the sum of the quantiles.
$\hookrightarrow$ A model is needed for the dependence structure.

Asymptotic Single Risk Factor (ASRF) model

The ASRF model

- is a default-mode (Merton-type) model proposed by Vasicek in 1991,
- has played a central role for its regulatory applications in the Basel Capital Accord Framework,
- is based on the following assumptions:

1. a unique systematic risk factor (single-factor model): economic risk
$\hookrightarrow$ the losses of the borrowers are correlated only through one systematic factor,
2. an infinitely granular portfolio (characterized by a large number of small size loans)
$\hookrightarrow$ diversification of the idiosyncratic risks, but not of the systematic risk,
3. a dependence structure described by a Gaussian copula
$\hookrightarrow$ the most important theoretical hypothesis,

- gives closed-form expressions for the expected and unexpected losses.

Asymptotic Single Risk Factor (ASRF) model

- The qth borrower defaults before time t if a latent variable $X_t^{(q)}$ (normalized asset) goes below a threshold value:

$$X_t^{(q)} = a^{(q)} Z_t + \sqrt{1-(a^{(q)})^2}\varepsilon_t^{(q)}$$

where
    - Z_t = systematic (economic) risk factor,
    - $\varepsilon_t^{(q)}$ = idiosyncratic factor,
    - $a^{(q)} = a_g$ factor loading (Basel: constant; here: depends on group).

Gaussian copula: $(Z_t, \varepsilon_t^{(1)}, \varepsilon_t^{(2)}, \dots)$ are i.i.d. standard Gaussian.
- The threshold values are obtained from the group-dependent unconditional migration matrices

$$z_{g,ij} = \Phi^{-1}\Big(\sum_{j'=j}^{K} (\mathbf{M}_g)_{ij'}\Big), \qquad \mathbb{P}\big(X_t^{(q)} \leq z_{g,ij}\big) = \Phi(z_{g,ij})$$

- The group-dependent conditional migration matrix is

$$\sum_{j'=j}^{K} (\mathbf{M}_g(Z_t))_{ij'} = \mathbb{P}\big(X_t^{(q)} \leq z_{g,ij} | Z_t\big) = \Phi\Big(\frac{z_{g,ij} - a_g Z_t}{\sqrt{1-(a_g)^2}}\Big)$$

Climate-Extended Risk Model (CERM) - principle

The Climate-extended model

- is a Multi-Factor Merton-type model,
- is based on the following assumptions:

1. several systematic risk factors (multi-factor model): economic, physical, transition risks,
2. an infinitely granular portfolio (characterized by a large number of small size loans),
3. a dependence structure described by a Gaussian copula,

- gives efficient Monte-Carlo estimations of the expected and unexpected losses.

Basic references:

- Vasicek Model
  Vasicek, O., The distribution of loan portfolio value, Risk, Dec. 2002.
- Multi-Factor Merton Model
  Pykhtin, M., Multi-factor adjustment, Risk, March 2004.

Climate-Extended Risk Model (CERM) - ingredients

Additional ingredients (compared to ASRF):

- Idiosyncratic risks, economic risk are stationary.
- Physical and transition risks evolve in time.

$\hookrightarrow$ *Climate scenarios are needed for the intensities of the systematic risk factors.*

- Physical risk factors can be regional.
- Systematic risk factors can be correlated.
  For instance, anti-correlation between economic and transition risks or correlation between regional physical risks.

$\hookrightarrow$ *Correlation structure between systematic risk factors is needed.*

- Expositions of borrowers to systematic risk factors (micro-correlations) may evolve in time (by mitigation strategies).

$\hookrightarrow$ *Micro-correlations w.r.t. systematic risk factors are needed for all groups.*

- The historical unconditional migration matrices are used at $t = 0$.

$\hookrightarrow$ *Same historical migration matrices as for ASRF model are needed.*

Note: The unconditional migration matrices evolve in time due to the non-stationarity of the physical and transition risks.

Climate-Extended Risk Model (CERM) - structure

The qth borrower defaults before time t if a latent variable $X_t^{(q)}$ (normalized asset) goes below a threshold value:

$$X_t^{(q)} = \boldsymbol{a}_t^{(q)} \cdot \boldsymbol{Z}_t + \sqrt{1 - \boldsymbol{a}_t^{(q)} \cdot \mathbf{C}\boldsymbol{a}_t^{(q)}}\,\varepsilon_t^{(q)}$$

where

$\boldsymbol{Z}_t =$ systematic risk factors (with correlation matrix $\mathbf{C}$),

$\varepsilon_t^{(q)} =$ idiosyncratic factor,

$\boldsymbol{a}_t^{(q)} =$ factor loadings; they are the products of time-dependent macro-correlations and time- and group-dependent micro-correlations.

- macro-correlations: intensities of the systematic risk factors, expressed in same units (impact to GDP growth rate for instance);
  - *constant* for economic risk;
  - given by *(IPCC) carbon emission pathway* for transition risk;
  - given by *(IPCC) GDP growth rate assessment* for physical risk.
- micro-correlations: expositions of borrowers to systematic risk factors;
  - given by *climate ratings*.

Climate-Extended Risk Model (CERM) - results

Conditional loss given the systematic risk factors $\boldsymbol{Z} = (\boldsymbol{Z}_1, \ldots, \boldsymbol{Z}_T)$:

$$L(\boldsymbol{Z}) = \sum_{t=1}^{T} L_t(\boldsymbol{Z})$$

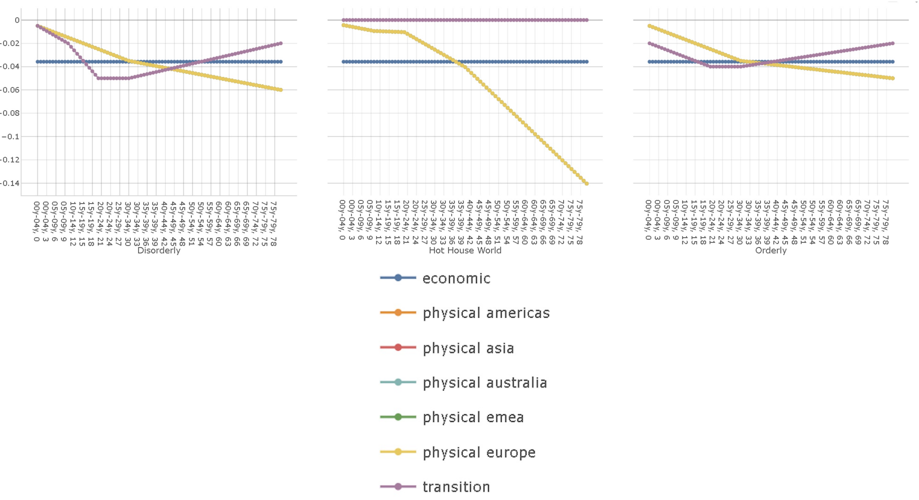
$$L_1(\boldsymbol{Z}) = \sum_{g=1}^{G} \sum_{i=1}^{K-1} (\boldsymbol{M}_{g,1}(\boldsymbol{Z}_1))_{iK} \mathrm{LGD}_{g,i,1}(\boldsymbol{Z}_1) \mathrm{EAD}_{g,i,1}$$

$$L_t(\boldsymbol{Z}) = \sum_{g=1}^{G} \sum_{i,j=1}^{K-1} \big(\boldsymbol{M}_{g,1}(\boldsymbol{Z}_1) \cdots \boldsymbol{M}_{g,t-1}(\boldsymbol{Z}_{t-1})\big)_{ij} (\boldsymbol{M}_{g,t}(\boldsymbol{Z}_t))_{jK} \mathrm{LGD}_{g,j,t}(\boldsymbol{Z}_t) \mathrm{EAD}_{g,i,t}$$

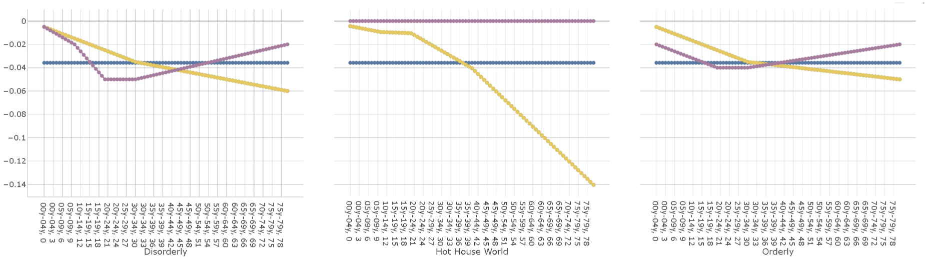
for $t \geq 2$. Here

- Explicit formulas are available for all terms.
- $L^{\mathrm{e}} = \mathbb{E}[L(\boldsymbol{Z})]$.
- L^{u} such that $\mathbb{P}(L(\boldsymbol{Z}) \leq L^{\mathrm{u}}) = 99.9\%$ (or 99% or 90%).
- Monte Carlo simulations can be carried out to estimate L^{u} or the distribution of $L(\boldsymbol{Z})$.
- Sensitivity indices (w.r.t. groups) can be estimated.

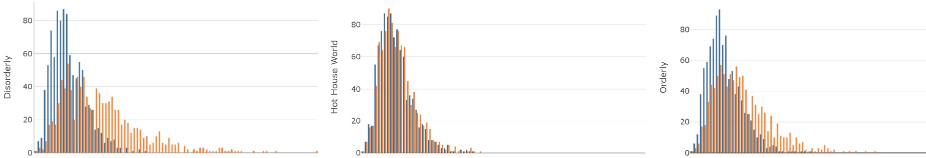
Three climate scenarios (macro-correlations) [IPCC]:



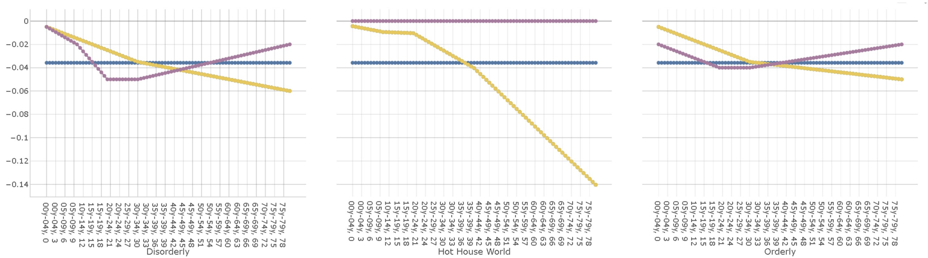
Climate-Extended Risk Model (CERM) - illustrations



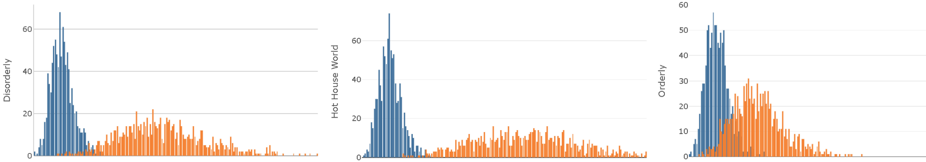
Loss distribution for time horizon $T = 2050$:



Blue: no physical/transition risk; orange: with physical/transition risks.



Loss distribution for time horizon $T = 2100$:



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Climate-Extended Risk Model (CERM) - summary

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