On Memory and Regimes in Economic Time Series: Introduction

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Josselin Garnier and Knut Sølna approach

Understanding price dynamics over various scales and how they change with time.

In finance, a follow-up of Mandelbrot fractional Brownian motion.

Relating auto-correlation of returns in [-1,1] to the Hurst exponent in [0,1].

So far

The analysis is:

1. Continous time
2. Univariate
3. Stationary

Extensions

Garnier and Sølna	Macroeconomics (short run): Stabilization policy, Monetary policy
Continuous time, large sample, large number of observations	Discrete time, short sample N=60 for a regime, with quarterly data
Univariate: exogenous autocorrelation changes	Bivariate: endogenous auto-correlation changes due to feedback-rule equation
Stationary time series	Some regimes where data are non-stationary with bubbles and crashes

From financial market data to monetary policy (macroeconomics)

1. Data are at monthly (inflation) or quarterly frequency (Gross domestric product): closer to discrete time than to continuous time with intraday or daily data for finance.

Does the frequency of measurement matter? Does discrete time versus continuous time matter in the analysis?

Endogenous auto-correlation breaks (exogenous correlation faces the Lucas critique (1976))

2. Inflation dynamics depend on two equations: a propagation mechanism and a feedback rule (for example, the Taylor rule). A regime change may be endogenous with positive-feedback passive policy versus negative-feedback active policy.

3. Bubbles versus quiet periods

Sometimes bubble, boom and crash regimes do not reject the unit root tests, for the auto-correlation = non-stationarity univariate time series.

In addition, bivariate correlations may change due to contagion during a crisis.

Other periods may lead to stationary time series variables. Quiet period regimes are sometimes related to stationary time series.

7 periods (inflation, federal funds rate, output gap)

1960-1972: auto-correlation (0.9, 0.9, 0.9).

1972-1979: (infl, output gap): **+0.2**

1985-2006: auto-correlations (**0.6**, 0.98, 0.96)

infl/gap: **0**, (output gap, fyff): **+0.5**;

before 1991: (inflation, fyff) **0.4**

1991-2006: (inflation, fyff): **-0.2**.

2007-2018: Auto-correlation of inflation = zero, auto-correlation of the Fed funds rate > 0.98

Adam Smith (1776)

turbances, he predicts, the system will come to rest in an equilibrium such that the market price will equal the natural price: "The natural price, therefore, is, as it were, the central price, to which the prices of all commodities are continually gravitating. Different accidents may sometimes keep them suspended a good deal above it, and sometimes force them down even somewhat below it. But whatever may be the obstacles which hinder them from settling in this center of repose and continuance, they are constantly tending towards it."[30] In the presence of disturbances, on the other hand, there will be certain "occasional and temporary fluctuations in the market price of any commodity,"[31] which accompany the stabilizing action of the system.

We estimated ρ, BUT closed-loop ρ= A+BF is subject to the Lucas critique (1976)

ECONOMETRIC POLICY EVALUATION: A CRITIQUE

Robert E. Lucas, Jr.

As observed in section 4, one cannot meaningfully discuss optimal decisions of agents under arbitrary sequences $\{x_t\}$ of future shocks. As an alternative characterization, then, let policies and other disturbances be viewed as stochastically disturbed functions of the state of the system, or (parametrically)

$$(16) \qquad x_t = G(y_t, \lambda, \eta_t)$$

where G is known, λ is a fixed parameter vector, and η_t a vector of disturbances. Then the remainder of the economy follows

$$(17) \qquad y_{t+1} = F(y_t, x_t, \theta(\lambda), \epsilon_t) ,$$

where, as indicated, the behavioral parameters θ vary systematically with the parameters λ governing policy and other "shocks". The econometric problem in this context is that of estimating the function $\theta(\lambda)$.

Fuhrer (2009), ECB working paper
Handbook of Monetary Economics

Perhaps the first test of persistence should be a unit root test. If inflation contains a unit root, its persistence is unquestionably large (infinite) and its variance is unbounded.[17] Many papers test for a unit root in inflation (see Barsky (1987), Ball and Cecchetti (1990)); prior to the 1990s, the results tend to suggest a unit root in inflation. In more recent years, researchers are more likely to be unable to reject stationarity. Most monetary models would suggest that the more vigorous attention to inflation on the part of central banks around the world in recent decades is responsible for this change.

the central bank. To make matters simple, consider the stylized, backward-looking model of inflation below

$$\begin{aligned} \pi_t &= \pi_{t-1} + a x_t \\ x_t &= -b f_t \\ f_t &= c \pi_t \end{aligned} \tag{2.8}$$

The first equation is a skeletal "Phillips curve," in which the change in inflation is positively related to a variable x_t, which we will take here to be the output gap. The output gap in turn depends negatively on the short-term policy rate f_t (for federal funds rate), and the policy rate is a positive function of inflation (with an implicit target inflation rate of 0). The solution for inflation is

$$\begin{aligned} \pi_t &= \alpha \pi_{t-1} \\ \alpha &\equiv \frac{1}{1+abc} \end{aligned} \tag{2.9}$$

Inflation will follow a first-order autoregression, and will be less persistent–the coefficient α will be smaller–the larger is the policy response to inflation (c), the more responsive is the output gap to the policy rate (b), and the more responsive is inflation to the output gap (a). In this simple framework, a central bank that behaves implicit target inflation rate of 0). The solution for inflation is

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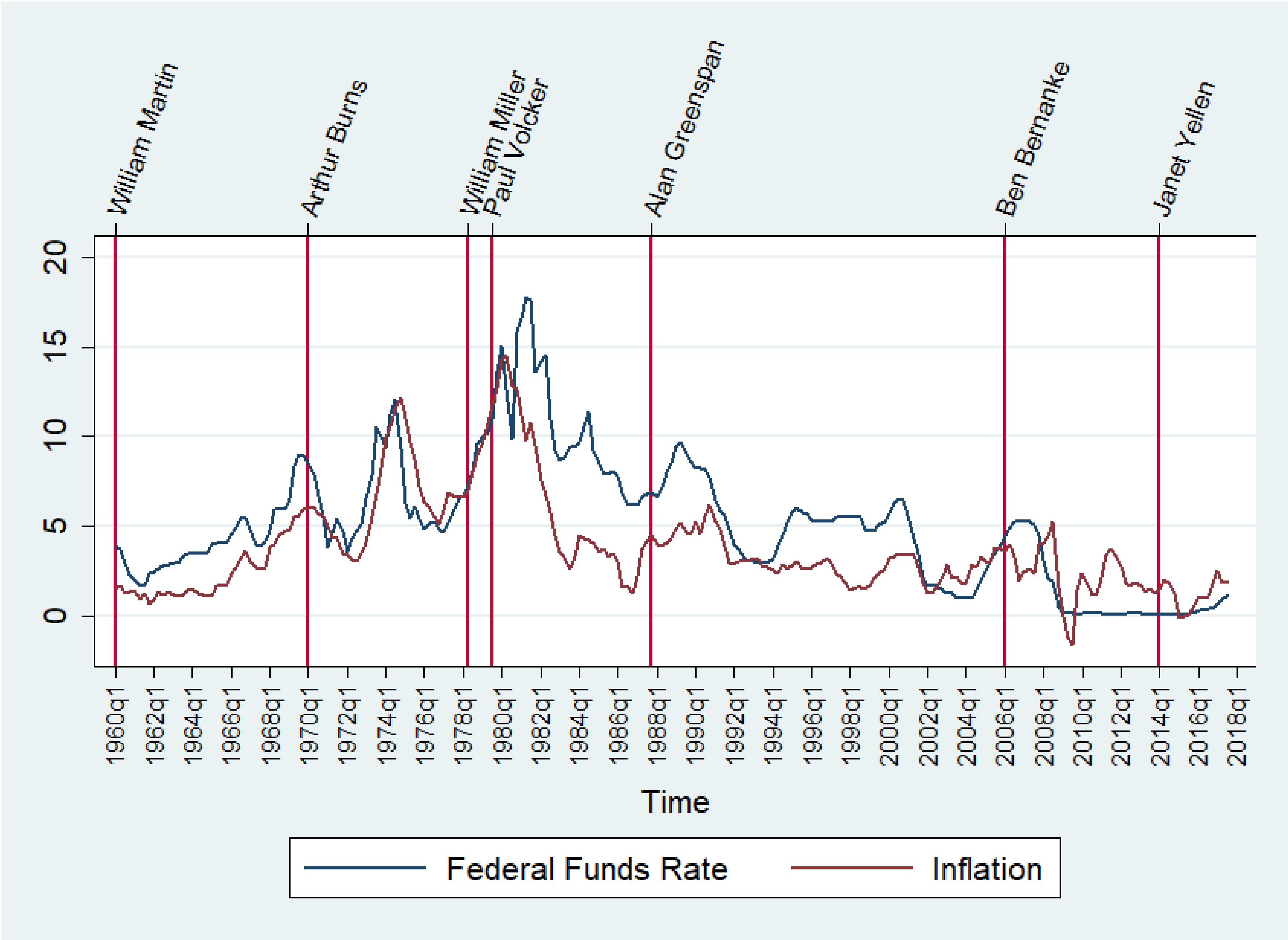
Inflation will follow a first-order autoregression, and will be less persistent–the coefficient α will be smaller–the larger is the policy response to inflation (c), the more responsive is the output gap to the policy rate (b), and the more responsive is inflation to the output gap (a). In this simple framework, a central bank that behaves more aggressively in moving inflation towards its target will reduce the persistence of inflation. The intuition from this skeletal model generalizes to a number of more sophisticated models that include rational expectations and a richer description of the key elements sketched above.[24]



William Martin
Arthur Burns
William Miller
Paul Volcker
Alan Greenspan
Ben Bernanke
Janet Yellen
0
5
10
15
20
1960q1
1962q1
1964q1
1966q1
1968q1
1970q1
1972q1
1974q1
1976q1
1978q1
1980q1
1982q1
1984q1
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1996q1
1998q1
2000q1
2002q1
2004q1
2006q1
2008q1
2010q1
2012q1
2014q1
2016q1
2018q1
Time
Federal Funds Rate
Inflation

1st order single input (interest rate, policy instrument), single output (inflation, policy target = inflation targeting: only one target)

(1) State law of motion, propagation mechanism **in deviation from the equilibrium, which is detrended**:

$$\pi(t+1) = A\,\pi(t) + B\,i(t).$$

(2) Feedback rule: $i(t) = F\,\pi(t)$

ONLY ONE DYNAMIC EQUATION WITH ONLY ONE EIGENVALUE

$\pi(t+1) = (A+BF)\,\pi(t).$

Auto-correlation of inflation falls = Volcker and Taylor principle effect

Auto-regression = closed-loop: B<0

$\pi(t+1) = (A + BF)\,\pi(t)$

ASSUME: A=1-B: effect of the real interest rate.

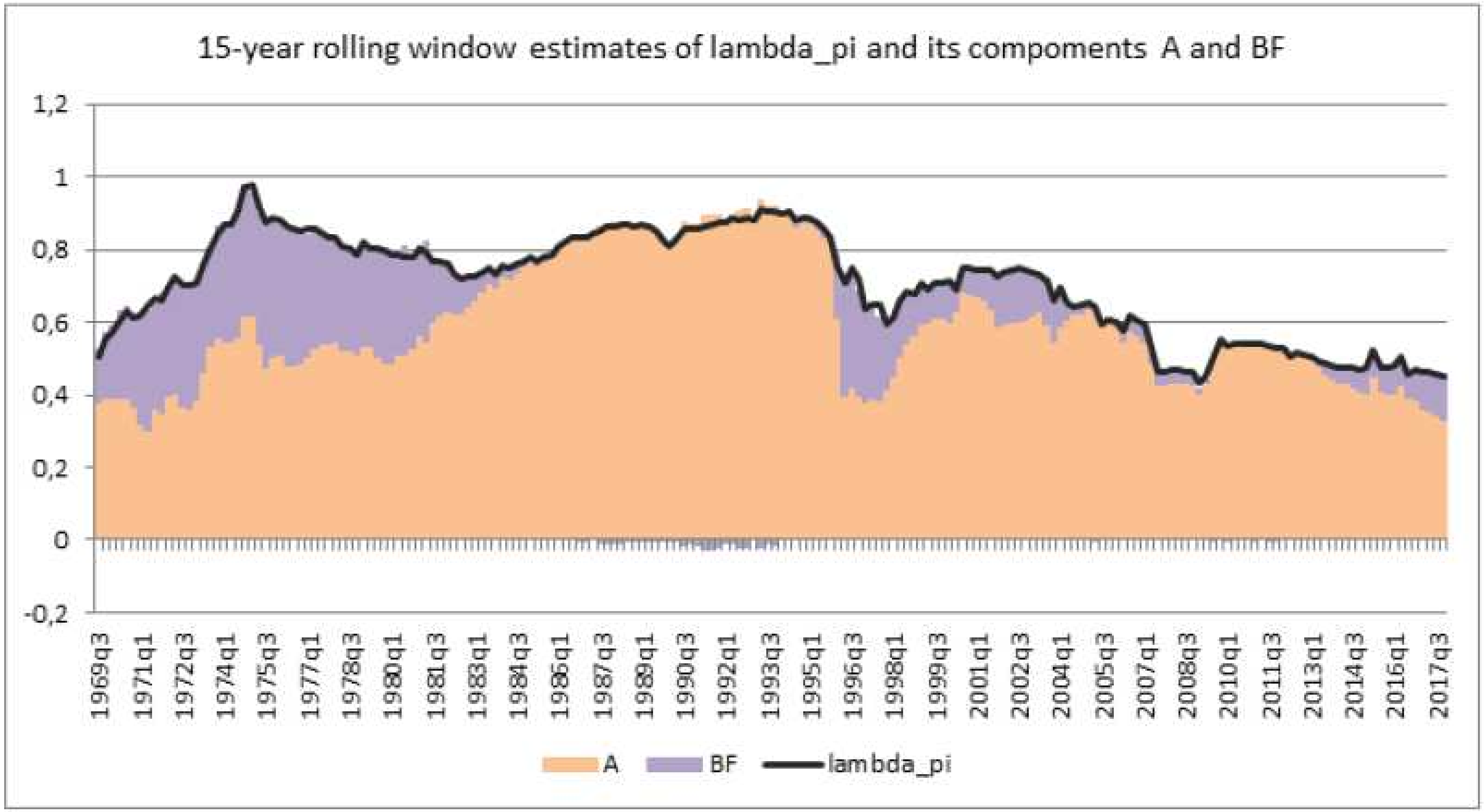
$\pi(t+1) = \pi(t) + B\,(i(t) - \pi(t))$

F>1 larger Volcker–Greenspan (negative feedback) than (1979-2006)

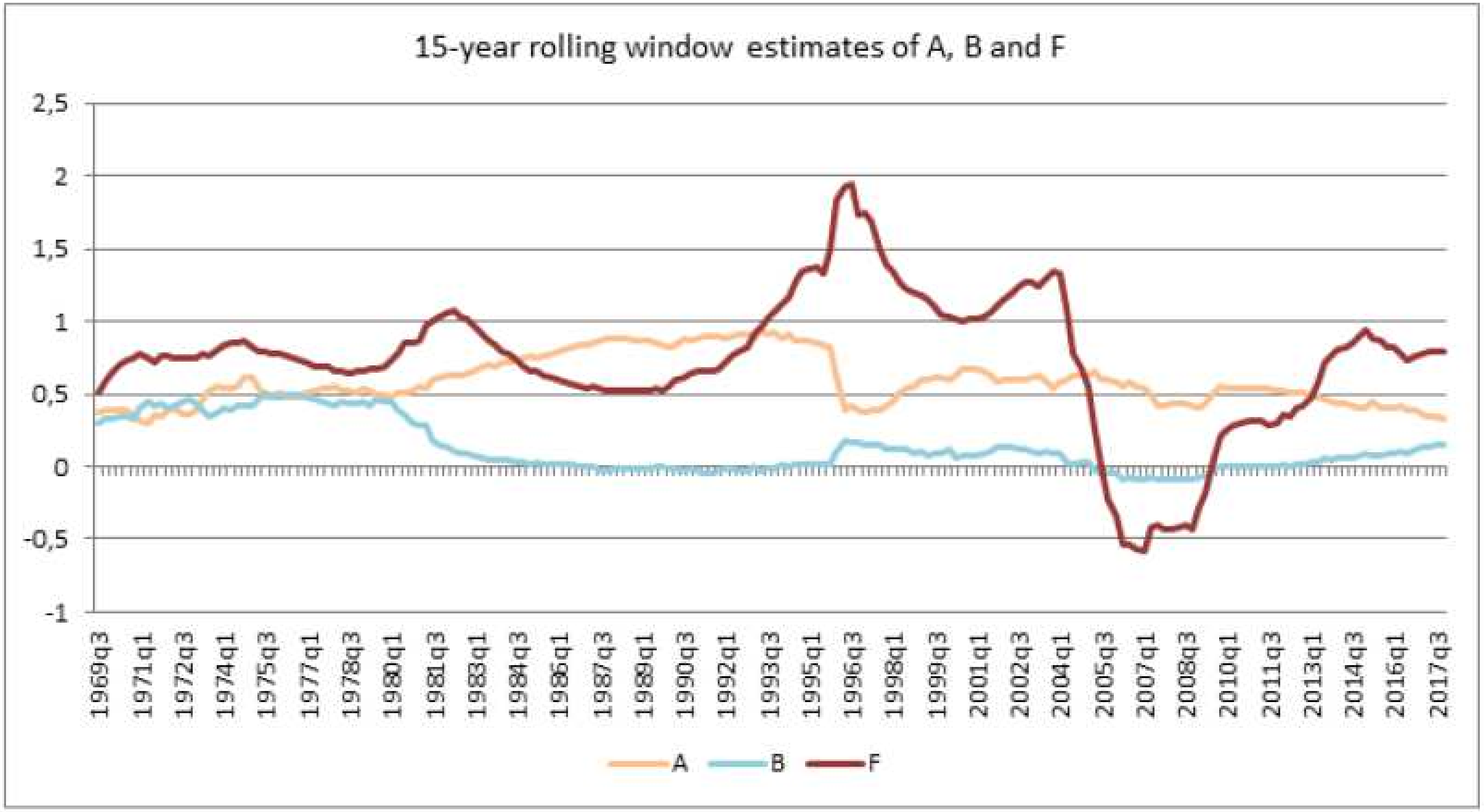
F<1 before 1979 Volcker (positive feedback)

(A + BF) Volcker–Greenspan <1< A+BF before Volcker (exploding inflation of the 1970s)

A brown + BF blue = A+BF auto-correlation



Taylor principle F (dark red) > 1
F should be positive, but 1992-2006?
B is blue, A is pink



Using estimates

One finds the relation between the simple correlation bivariate parameter and the estimates A+BF

$\rho = 0.6 = A + B F = 0.3 + \mathbf{0.2} * \mathbf{1.5} = 0.3 + 0.3$

B>0 and F>0 instead of B**<0** and F>0.

BF is positive = it is “adding” auto-correlation as with a positive feedback.

Increasing the variance of the Fed funds rate (1979-1982) increases F>1

$i_t = F_\pi \pi_t + \varepsilon_t$ OLS simple regression

$$F_\pi = \beta = r_{i\pi}\frac{\sigma_i}{\sigma_\pi} = r_{i\pi}\frac{\sigma_i}{\sigma_\pi} > 1 \Leftrightarrow$$

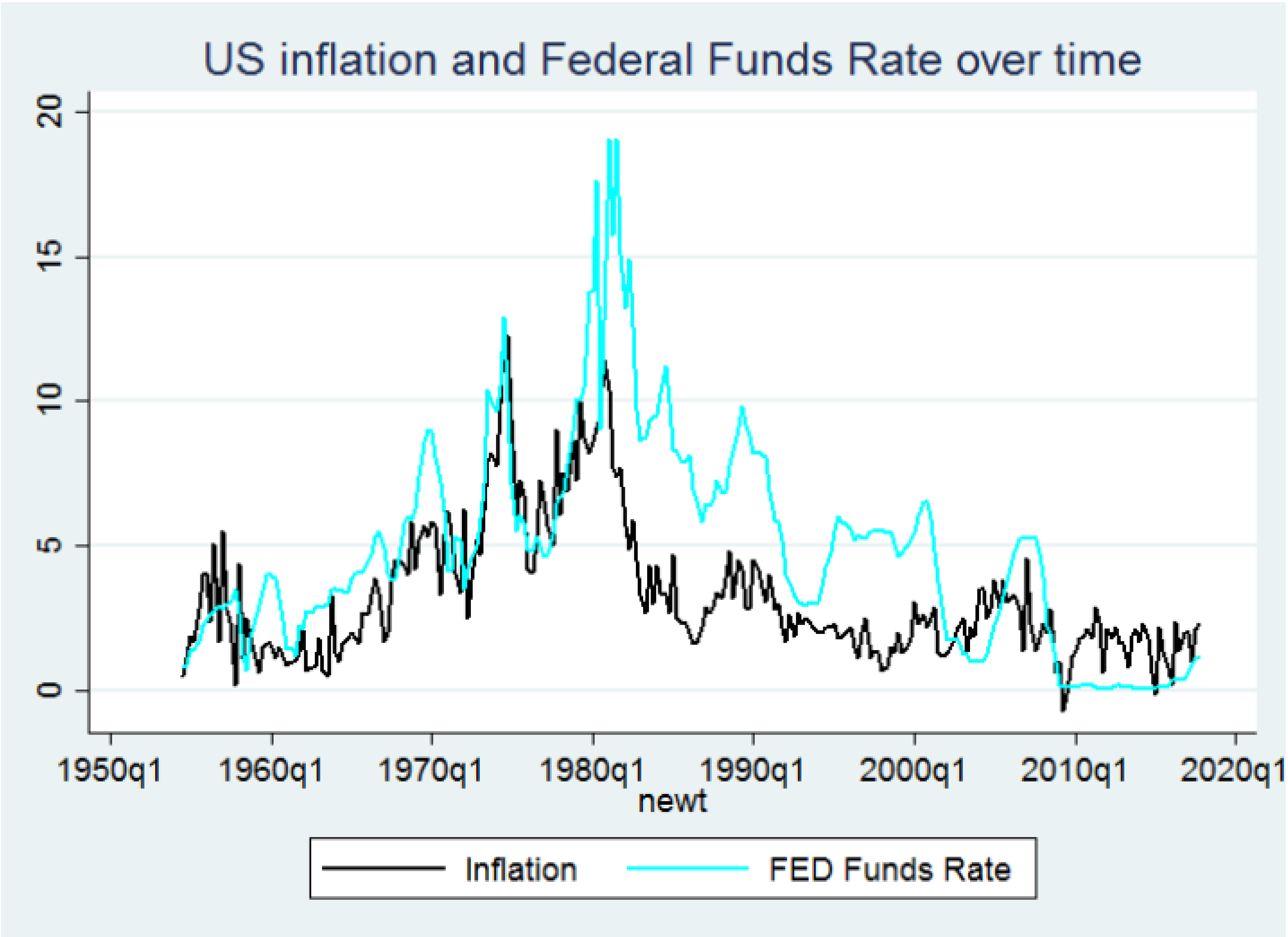
$$\frac{\sigma_i}{\sigma_\pi} = \frac{3.76}{2.03} = 1.85 > \frac{1}{r_{i\pi}} = \frac{1}{0.75} = 1.33 \text{ after 79q3}$$

$$F_\pi = 0.83 \cdot \frac{2.42}{2.71} = 0.74 \text{ before 79q3}$$

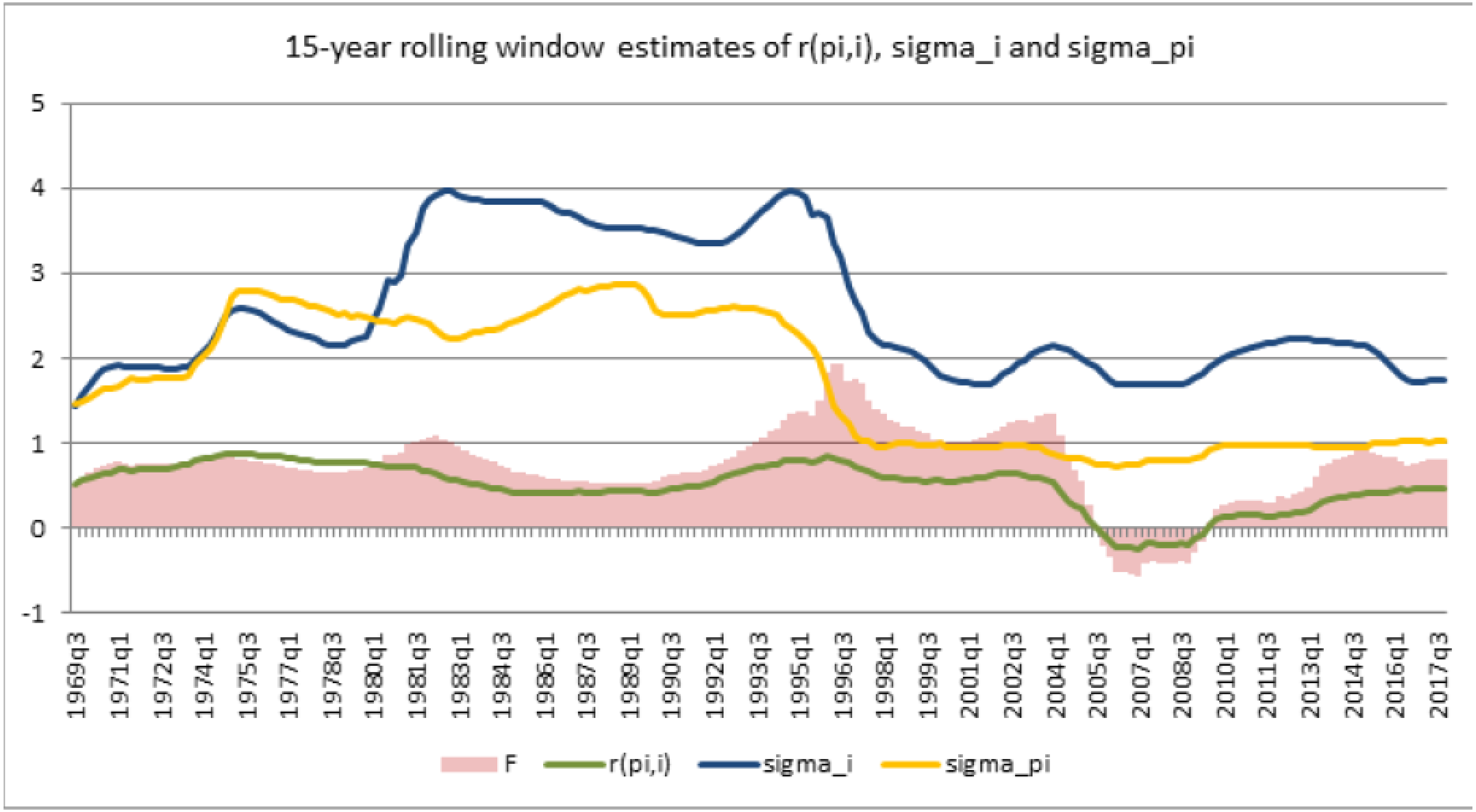
$$F_\pi = 0.75 \cdot \frac{3.76}{2.03} = 1.38 \text{ after 79q3}$$

$$R^2 = r_{i\pi}^2 = 0.83^2 = 0.69 \text{ before 79q3}$$

Check: variance of Fed funds rate



F=r*s(i)/s(pi)


15-year rolling window estimates of r(pi,i), sigma_i and sigma_pi
5
4
3
2
1
0
-1
1969q3
1971q1
1972q3
1974q1
1975q3
1977q1
1978q3
1980q1
1981q3
1983q1
1984q3
1986q1
1987q3
1989q1
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1999q3
2001q1
2002q3
2004q1
2005q3
2007q1
2008q3
2010q1
2011q3
2013q1
2014q3
2016q1
2017q3
F
r(pi,i)
sigma_i
sigma_pi

Exogenous univariate auto-correlation versus endogenous bivariate feedback correlation

TABLE 8 - $\lambda_\pi = A + BF$

Period	λ_π	A	B	F	$A + BF$	$A + BF - \lambda_\pi$
1954q3-1991q4	0.8497	0.7817	0.0736	0.9236	0.8497	0.0000
1992q1-2006q4	0.5967	0.5432	-0.0948	-0.5863	0.5988	0.0021
2007q3-2017q4	0.2367	0.1312	0.1707	0.6431	0.2410	0.0043

BF>0, A<A+BF<1

Period	Parameter	Obs.	coefficient	Sdt. dev.	t-stat
1954q3-1991q4	A	149	0.7817	0.0566	13.81 ***
	B	149	0.0736	0.0405	1.81 *
	F	150	0.9236	0.0859	10.75 ***
	ρ_π	148	-0.2343	0.0806	-2.91 ***
	ρ_i	149	0.7797	0.0514	15.17 ***
1992q1-2006q4	A	60	0.5432	0.1073	5.06 ***
	B	60	-0.0948	0.0466	-2.04 **
	F	60	-0.5863	0.2900	-2.02 **
	ρ_π	59	-0.1341	0.1343	-1.00
	ρ_i	59	0.9380	0.0467	20.07 ***
2007q3-2017q4	A	44	0.1312	0.1545	0.85
	B	44	0.1707	0.0841	2.03 **
	F	44	0.6431	0.2396	2.68 ***
	ρ_π	43	0.0922	0.1425	0.65
	ρ_i	43	0.8368	0.0747	11.21 ***

Significance levels : * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

On the same line, a cross-correlation does not change signs.

TABLE 6 - Cross-correlation coefficients

Period	Cross-correlation	π_{t-2}	π_{t-1}	π_t	π_{t+1}	π_{t+2}
1954q3-1991q4	Fed. funds rate i_t	0.6737	0.6732	0.6623	0.6232	0.5714
1992q1-2006q4	Fed. funds rate i_t	-0.1088	-0.1600	-0.2566	-0.2744	-0.3729
2007q3-2017q4	Fed. funds rate i_t	0.3801	0.3357	0.3827	0.2007	0.1356

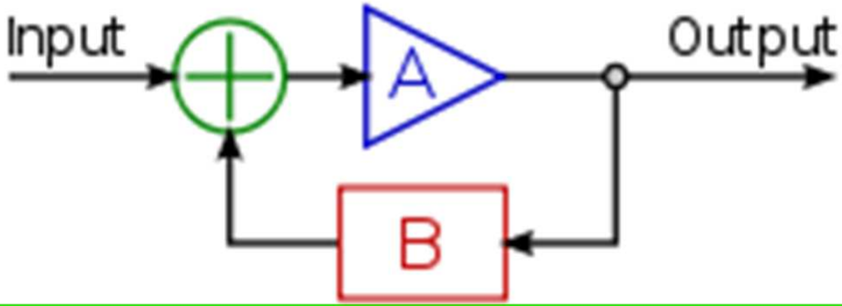
Cf. supply and demand

$B < 0$: victory of the transmission mechanism

$F < 0$: defeat of the feedback-rule intuition

$B > 0$: defeat of the transmission mechanism intuition

$F > 0$, $F > 1$: victory of the feedback-rule intuition



(1) State law of motion, propagation mechanism **in deviation from the equilibrium, which is detrended**:

$$\pi(t+1) = A\,\pi(t) + B\,i(t).$$

(2) Feedback rule: $i(t) = F\,\pi(t)$

Negative-feedback sign restrictions: B<0, F>0

BUT: $Cov(\pi(t+1), i(t))<0$ and $cov(i(t), \pi(t))>0$

Sign flip of one period ahead, cross-correlation very unlikely!

Identification problem in simult. equation models

Assume form of the equations correctly represents the 'true' causal model, and all variables are observable, but parameters are unknown.

How to infer them from data? **Not always possible**

Simple Supply-Demand Model
(cf. Supply - Demand situation faced by early econometricians, Morgan 1990)

$$q = \alpha_1 p + \varepsilon_1 \quad \ldots \quad \text{demand}$$
$$q = \alpha_2 p + \varepsilon_2 \quad \ldots \quad \text{supply}$$

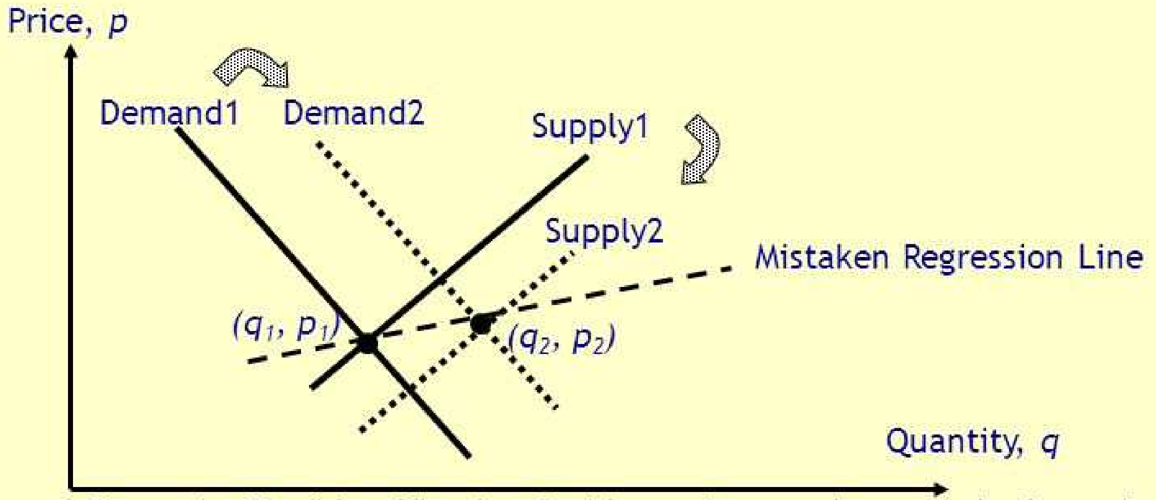


Figure 1 - The Identification Problem - Causes change in both mechanisms

$q = a_1 + \mathbf{b_1} p + c_1 \mathbf{y} + u_1$ demand function $\mathbf{b_1 < 0}$

$q = a_2 + b_2 p + c_2 \mathbf{R} + u_2$ supply function $\mathbf{b_2 > 0}$

$$q = \frac{a_1 b_2 - a_2 b_1}{b_2 - b_1} + \frac{c_1 b_2}{b_2 - b_1} y - \frac{c_2 b_1}{b_2 - b_1} R + \text{an error}$$

$$p = \frac{a_1 - a_2}{b_2 - b_1} + \frac{c_1}{b_2 - b_1} y - \frac{c_2}{b_2 - b_1} R + \text{an error} \qquad (9.3)$$

$$q = \pi_1 + \pi_2 y + \pi_3 R + v_1$$

$$p = \pi_4 + \pi_5 y + \pi_6 R + v_2$$

$$\pi_1 = \frac{a_1 b_2 - a_2 b_1}{b_2 - b_1}, \quad \pi_2 = \frac{c_1 b_2}{b_2 - b_1} \quad \text{etc.}$$

$$\hat{b}_1 = \frac{\hat{\pi}_3}{\hat{\pi}_6} \quad \hat{b}_2 = \frac{\hat{\pi}_2}{\hat{\pi}_5}$$

$$\hat{c}_2 = \hat{\pi}_6(\hat{b}_1 - \hat{b}_2) \quad \hat{c}_1 = -\hat{\pi}_5(\hat{b}_1 - \hat{b}_2)$$

$$\hat{a}_1 = \hat{\pi}_1 - \hat{b}_1 \hat{\pi}_4 \quad \hat{a}_2 = \hat{\pi}_1 - \hat{b}_2 \hat{\pi}_4$$

A thorny negative-feedback identification issue

Identification issue: One needs a variable that matters in one of the equations but not the other one.

Feedback rule: all the indicators (regressors) of the drivers of inflation may be taken into account by Central Banks (why would they omit one observable indicator?).

With a single inflation targeting mandate, there are no outside variables.

A dual or triple mandate (output gap, financial stability) may leave open additional components besides drivers of inflation in the policy rule = potential for identification (if no coincidence).